Chapter 3 continues discussing inverse functions. In this section, we will learn what a one-to-one function is, how inverse functions are special cases of modified one-to-one functions, and what logarithms are, particularly, the natural logarithm.

# One-to-One Functions

****One-to-one function (object) – a function such that whenever .

If a function is one-to-one, then, its derivative will always be greater than 0 where it is defined.

Horizontal line test (method) – a test of whether a function is one-to-one or not. See Figure 1. Procedure:

Figure

1. Spread a horizontal line (such as a pencil) across the function.
2. Move the line all the way up.
3. Move the line all the way down.
4. If the line never intersected the curve more than once at any time, it is a one-to-one function.

# Inverse Functions

****Inverse function (object) – a one-to-one function such that if and only if .

**Caution**: does not have an exponent. in general.

## Domain and Range

Since and share the same ’s and ’s, their domain and range are closely related. See Figure 2.

* The domain of is the range of .
* The domain of is the range of .

Figure

## Cancellation Equations

By substitution, we find that, for all in the appropriate domain,

* for all in the domain of .
* for all in the domain of .

## How to Make an Inverse Equation

The textbook recommends this procedure[[1]](#footnote-1) for making inverse equations:

1. Write .
2. Solve an equation for in terms of .
3. Swap and .
4. .

## Graphical Perspective

Figure

Looking at any one-to-one function that is real, the inverse can be found by mirroring the original function across the line. See Figure 3.

# Calculus of Inverse Functions

Inverse functions can have properties of calculus applied to them too, just like “regular” functions.

## Continuity

If a one-to-one function is continuous on an interval ,

* the inverse function is continuous on the interval .

## Derivatives

If and ,

# Logarithmic Functions

For all where , all exponential functions are continuous on and one-to-one.

* Therefore, an inverse exists, , the logarithmic function.

Logarithmic function, logarithm (object) – the inverse of an exponential function, denoted . Figure 4 shows an example logarithmic function.

Figure

**Pronunciation**: can be pronounced:

* “the base- log of ”
* “the base- logarithm of ”
* “the log of to base ”
* “the logarithm of to base ”

## Inverse Function

If , then , and vice versa.

## Cancellation Equations

* .
* .

## Domain and Range

If ,

* The domain
* The range .

## Laws of Logarithms

* (where )

# Natural Logarithm

Natural Logarithm (object) – the logarithm with base . Notated .

## Cancellation Equations

## Change of Base Formula

If we want to take the value of a logarithm with a base the calculator doesn’t work with (some calculators only do base-10 and base-e logarithms), the change of base formula can be used.

## Limits at Infinity

# What Did You Learn?

* What is a one-to-one function? How does the horizontal line test verify a function being one-to-one?
* How do you make an inverse function from another function? Why are all inverse functions one-to-one?
* What is a logarithm? How is used in logarithms?

1. Stewart, J. (2013). Essential Calculus - Early Transcendentals. In J. Stewart, *Essential Calculus - Early Transcendentals* (p. 131). Belmont, CA: BROOKS/COLE CENGAGE Learning. [↑](#footnote-ref-1)